



More Trouble With Math

Chapter 3: What the curriculum asks pupils to do and where the difficulties may occur.

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By Steve Chinn

More trouble with maths

INTRODUCTION

What the curriculum asks pupils to do and where difficulties may occur.

In the last chapter I looked at factors, for example, poor working memory, which could create learning difficulties for mathematics in the classroom. In this chapter I am looking at curriculum topics in mathematics and how they could create difficulties in learning.

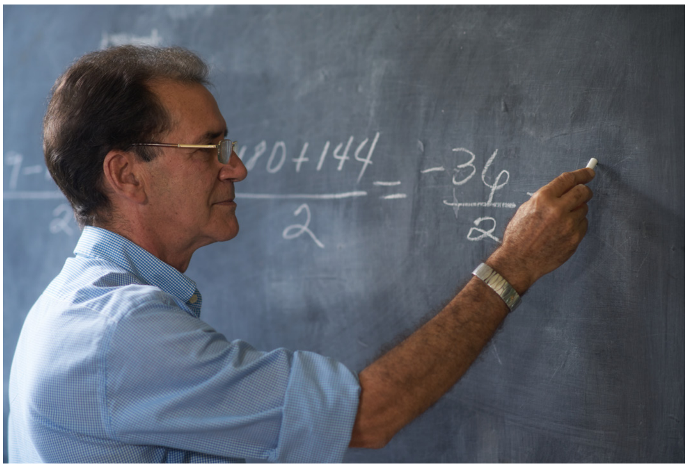
If we use a definition of 'curriculum' as 'a planned sequence of instruction', it seems to me that maths curricula are not going to be radically different, whether for the UK, for Singapore, for the USA or any country. Basic maths is basic maths. For example, subtraction has to precede division and division should precede fractions. If we take a broader definition of curriculum, that is as 'the totality of student experiences that occur in the educational process' and we interpret that as to include the way the programme is delivered, the style of the curriculum, then there is the potential for wide variation. Most of the on-going initiatives from the UK Government fit this latter interpretation of curriculum. Students receive a 'totality of experiences.' Not least in significance of these variations for learners is the pace of the curriculum.

We teachers tend to be as autonomous as possible within our own classrooms. It is hard for any authority to control that situation completely, despite rigorous regimes of inspection. The consequence is that any style or philosophy or pedagogy will be subject to significantly different interpretations.

As a first example of a style that might have unintended consequences I will select 'mastery'. So much depends on how 'mastery' is defined. My Oxford dictionary defines it as 'outstanding skill or expertise' and 'complete power and control'. It's a tad like the word 'excellence'. Both words leave small margins for interpretation. Historically, educators such as Bloom and Lindsley interpreted 'mastery' to mean the breaking down of a learning goal into a number of small learning objectives. This may also have been referred to as 'precision teaching' if those learning objectives are very detailed and precise. The pupil masters a small step and then moves to another small step.

The mastery model, as with all models, will work for some children, but not for all. There may be some variations in the way it is sometimes interpreted. There may be unintended consequences. For example, in England, the National Centre for Excellence in the Teaching of Mathematics states:

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. When to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage.



It's what is meant by 'the majority of pupils' and what percentage are in the minority. The normal distribution puts around 68% of pupils in the average band for whatever skill or sub-skill is being considered, and thus around 32% outside. And I am ever wary of 'expectations.'

If we, for example, wait for some children to have complete and accurate retrieval of all times table facts, especially if we rely predominantly on rote learning, then we will have a long wait for readiness to progress. In Chapter 9, I suggest the use of mathematical strategies as an alternative, or supportive, approach to this particular goal.

My second example is mental arithmetic. If the curriculum places great emphasis on this skill then, without suitable differentiation, some children will fail. Children who have poor short-term memories will find it impossible to remember, and therefore answer, questions that exceed their stm capacity. Children who have low capacity working memories will find it impossible to achieve an answer if the number of steps to reach that answer exceeds their WM capacity.

This chapter highlights a range of key topics, predicts where difficulties and confusions will arise and outlines some possible solutions. Some of these are dealt with in more detail in other chapters, but to avoid the need to flick between pages, I have kept the content somewhat self-contained. The chapter illustrates an analysis of content in terms of its likely interactions with learners. A similar analysis could be applied to any programme of work. This is the preventative medicine. Whatever the curriculum, the analogy I like is to compare preparing a mathematics lesson to preparing an expedition. You prepare for all the many problems you know you are going to encounter, and experience helps you to predict what they will be, but then experience tells you that there will still be some problems you will not have predicted. The curriculum is the basic guide to the journey. This chapter is the guide to some of the unpredicted events that may prevent your learners from having a successful expedition.

Throughout this chapter I will give some data extracted from my 15-minute norm-referenced maths test as used in the companion book on diagnosis, 'More Trouble with Maths'.

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1. Place value

Place value is a basic and fundamentally important topic which sets the foundations for success in numeracy. A deep understanding of place value is essential for many future topics in maths. It should not be assumed that pupils will generate links by osmosis. Many of the links will need explicit teaching. Also, it is easy to assume that an ability for recalling memorised information means that the information is therefore understood.

Most of the work on place value is within the pupils' previous experiences, the experiences that pre-date school. This is both a positive and negative influence. It is positive in the sense that pupils are working from familiar facts and awareness and should feel comfortable and confident, but negative in that they may already have formed some incorrect ideas and perceptions or maybe feel that because the work is familiar they do not need to consolidate and inter-relate their existing fact base to new work. Early experiences are of paramount importance.

'They come to formal education with a range of prior knowledge, skills, beliefs and concepts that significantly influence what they notice about the environment and how they organize and interpret it. This, in turn, affects their abilities to remember, reason, solve problems and acquire new knowledge.' ('How People Learn.' National Research Council. USA. 2000)1

Place value is a key concept in maths. Children who fail to grasp the idea of place value will find numeracy difficult and in the later stages of maths will make errors such as:

$$\begin{array}{r} 45 \\ +88 \\ \hline 1213 \end{array} \qquad \begin{array}{r} 45 \\ +88 \\ \hline 123 \end{array} \qquad \begin{array}{r} 45 \\ \times 22 \\ \hline 90 \\ 90 \\ \hline 180 \end{array} \qquad \begin{array}{r} 45 \\ 10\overline{)4050} \end{array}$$

These errors are serious in that they are rooted in a misunderstanding of this fundamental and influential concept, a concept that is at the heart of our number system. Such errors are a clear indication of the need to track back to the very basics of maths as your starting point for intervention.

Recognising errors is diagnostic. Marking answers wrong without that diagnosis is judgemental.

Misconceptions about place value can arise from early experiences. For example, we introduce numbers as 1, 2, 3, 4, 5 ... where the sequence 'gets bigger' as we track to the right. When place value arises, as with, say 24, the digit to the left is 'bigger'. Any confusion may be exacerbated by the previous experience of 4 being 'bigger' than 2. Such apparent inconsistencies in maths can derail an insecure learner. Their prior experiences and knowledge from outside school may add to the confusion. If the Hindu-Arabic system of number, which emphasizes place value, is to be understood with all its mathematical implications, then it should be taught thoroughly, with images and/or materials to support the concept (see Chapter 5). We may forget how sophisticated this concept is and how relatively recently it came to Western cultures. I looked at the brass plaque on an old bridge in Bath (my home town) UK. It was built in MDCCCLXXXVIII. Roman numerals are not easy to work with! (It was built in 1888).

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Addition and subtraction, especially involving crossing the tens, hundreds, thousands, etc will also be handicapped by a poor understanding of place value. Crossing the decades and hundreds is part of place value and becomes an essential part of addition and subtraction. Equally multiplication, by both the traditional methods and the 'chunking' methods, requires that place values are understood, so that, for example when multiplying by 45 the pupil needs to know and understand that the 4 is 40 and what effect this has on the multiplication. Pupils need to be able to break down numbers and that involves understanding place value. (Breaking down numbers can include using other number relationships as well as place value, for example 50 as 100 ÷ 2 or 99 as 100 -1 or 40 as 2 x 20).

Manipulatives and manipulatives (see Chapter 9) can be helpful in explaining the concept of place value, particularly base ten blocks and coins which can give a visual image to the symbolic representations of numbers and the base ten system. Numbers which involve zero (such as 1004) often need extra explanation and base ten materials and place value columns may help. Try working from 1444, through 1044 to 1004 using the base ten blocks and place value columns. Then take away the scaffolding in a way that suits the learner. Multiplying and dividing by 10 and powers of 10 may also help to show how the position of a digit in a number affects its value. My work on standardizing the 15-minute screener test for 'More Trouble with Maths' revealed how poorly this aspect of the base ten concept, particularly dividing by 100, 1000 and so on, is understood and carried out.

Data from the 15-minute test: two examples that require an understanding of place value. The data is for correct answers

$10\overline{)6030}$	10y	44.5%	13y	48.7%
23÷ 1000	10y	14.5%	13y	36.4%

2. Estimation and rounding and place value

Try work on placing a number in the correct position on number lines, both empty and full number lines. Work on rounding and on linking estimation to accurate computations leads pupils to learn the skills of over-viewing and checking problems. Estimation is a holistic skill and should be taught to complement the procedural skills of written arithmetic, even though pupils may show a marked preference towards just one of these skills (see Chapter 4). Remember that estimation is not precise, and is not meant to be precise, and that the required level of 'accuracy' of the estimation often depends on the particular situation, especially in 'rea-life'. The 'empty' number line is a good visual for practising this skill. (see Chapter 9 for more on manipulatives)

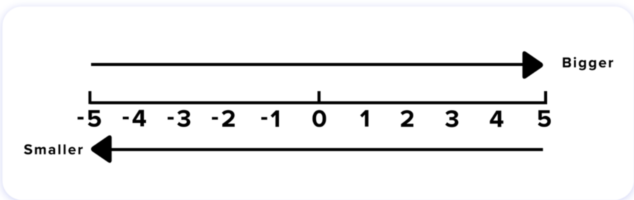
I have a concern that many students do not evaluate their answers, especially those with an inchworm cognitive style (see Chapter 4). The (2001) definition of dyscalculia used in the UK noted that, 'Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.' A survey of 391 special education professionals in the USA (Bryant et al, 2000)2 identifying characteristic behaviours of students with teacher-identified maths weaknesses puts "reaches 'unreasonable' answers" ninth out of thirty –three behaviours.

Rounding is a strategy that strengthens the skill of estimation in the sense of specifying 'levels', such as rounding to the nearest ten, nearest hundred, nearest thousand. It is also a good real-life skill, particularly useful for shopping. (Why is it, that after all the maths education delivered across the world, shops still price items as £4.95 or $47.99 or €999 and many shoppers persist in interpreting these as £4 and 47 and significantly under €1000? Maybe it is the power of the first digit you say?) From the data for the 15-minute test, rounding 551 to the nearest 100 was answered correctly by 90% of 13-year old pupils. Shops should take note of this high level of success.

3. Negative numbers

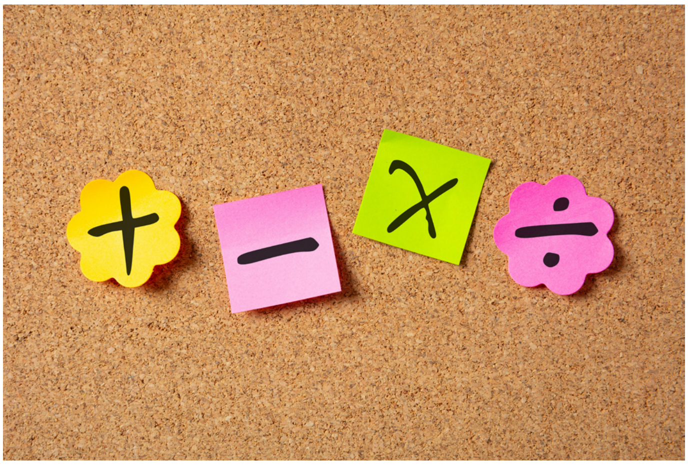
Negative numbers are quite challenging to learners' everyday experiences. Many learners find counting backwards in positive numbers problematic, so counting back in negative numbers is going to be a big challenge to their perception of consistency. So, it is good to use some real-life examples to help in explaining the concept. Temperature and lifts ('going up' and 'going down') are real-life examples where negative numbers occur. Both these examples are a vertical representation of a number line and thus not the familiar horizontal form. Consistency is challenged again, but awareness helps!

So, the quite sophisticated sequence on a number line is:



This sequence takes the counting backwards skill beyond the zero. The rules of the positive number sequence seem now to be reversed. In positive numbers 4 is bigger than 3. In negative numbers -4 is smaller than -3. In the first experiences of negative numbers these seeming inconsistencies can confuse uncertain learners and will need clear and explicit teaching. Consistency in the interpretation of number sequences is challenged again (and will be challenged yet again with fractions). Concepts that challenge previous learning often need materials and visual images to support an understanding of the symbols.

Introductory work on adding to a negative number (-3o C, warming up by 4o C or starting 3 floors below ground and taking the lift up 4 floors) and subtracting from a negative number (-3o C and cooling down by a further 3o C to – 6o C) sets the foundation to work from an image that is 'real' to an abstract and thus to symbolic representation (-3 + 4 = 1 and -3 - 3 = - 6). This topic also re-visits the idea that addition and subtraction are opposite versions of the same idea. The vertical number line may help illustrate the concept and the processes and act as an intermediate stage to the horizontal version. A vertical number line also gives meaning to up and down. (On a cold day, say -8 oC, if I could take away -7 oC of those degrees, that is, make the temperature warm up by 7 oC), then the temperature would be -1 oC. The mantra is, 'Minus a minus makes a positive change').



4. Calculations and computations

Rapid recall of addition and subtraction facts

There will be some pupils for whom the task of recalling from memory the so-called 'basic' facts will be difficult. This issue is frequently quoted as one of the key characteristics of dyscalculia in the research. The additional pressure of having to respond quickly will exacerbate the problem. It will be vital for these pupils, and helpful for the others if the connections and patterns are explained. This has been an issue for many years for many learners. Low achievers in maths often rely on counting in ones, rather than recall and/or linking facts. The use of non-counting strategies, such as adding 9 by adding 10 then subtracting 1, is far more prevalent in high achievers. If these strategies are performed quickly teachers may interpret these processes as retrieval. They are more cognitive than that.

One of the great pluses of maths is that facts can be linked. This is much less evident in other subjects. For example, if I know that the capital city of France is Paris, it does not help me know the name of the capital city of Bulgaria. If I know that 5 + 5 = 10 then I can work out many things, for example that 5% is half of 10% or that 5 + 6 = 11.

'How People Learn' (2000, National Academy of Sciences, USA)1 identifies three key findings for learning. The second of these principles is:

To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

This is particularly apposite at so many levels for students with difficulties in learning maths. The parts (b) and (c) support (a) for these pupils especially. For this population a key question is what constitutes 'a deep foundation of factual knowledge' and does this only infer straight retrieval from long-term memory? I hope that (b) and (c) suggest the answer is 'No.'

However, even pupils with poor memories for basic facts will need to learn some facts (a) in order to use strategies and make links (b and c). The question is, 'Which are the most useful facts?' For addition and subtraction these facts are likely to be the doubles and the number bonds for ten. The most useful facts are, not surprisingly, the ones that can be extended and used to access as many other facts as is possible. The inter-relationships of numbers and operations will help students to build strategies which not only help them retrieve more facts, but also support their understanding of basic number and operation concepts. In terms of teaching strategies to students, it is important to realize that all pupils will benefit from learning these links. This is truly inclusive teaching. It is the low achieving students that tend not to develop strategies for themselves. They need guidance. The more able students may not extract all the generalisations and benefits for understanding concepts beyond addition and subtraction, so guidance may help them, too. Strategies can explain why 6 x 7 = 42 rather than, say, 6 x 7 = 48.

As an example, the key facts, the number bonds for 10 can be extended to the two-digit equivalents such as 60 + 40 = 100 and then to 62 + 38 = 100. This involves understanding place value, although it also can extend and support existing levels of understanding.

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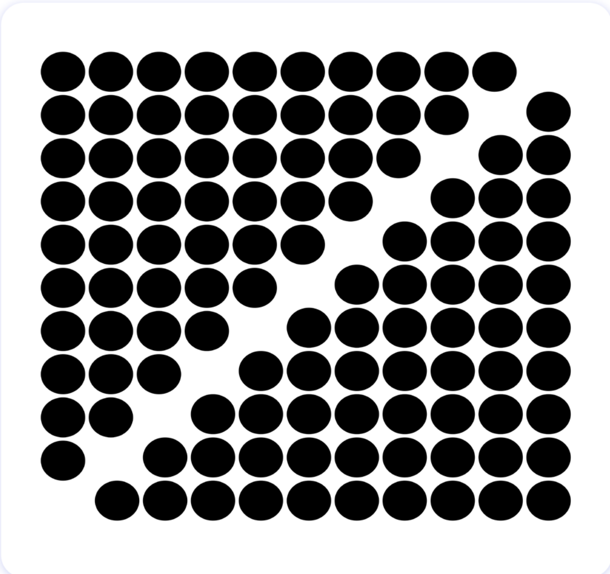
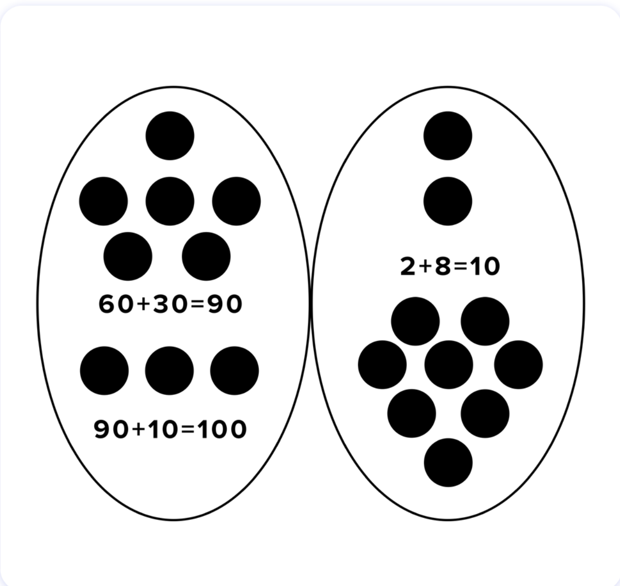
This progression may require good visual images, probably based on base ten materials or money. A pattern can, once more, be used.



The extension to a more complex pairing such as 62 + 38 could be seen as in two parts, tens digits which add to 90 (done with nine 10p coins) and ones-digits (using ten 1p coins) which add to 10. The two groups of coins add to 100p, £1



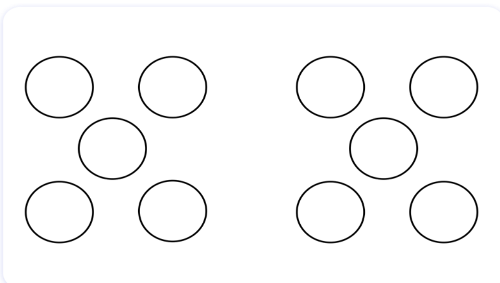
A second extension is to 1000, as with 850 + 150 which can be seen as two hundred digits, 8 and 1, which add to 9 hundred and two tens digits, 5 and 5, which add to 10 tens, that is 100. And 900 + 100 = 1000. More place value content.

Extension of the doubles is a strategy used by many pupils and can be taught and organised to be consistent. For example, doubles plus one may be used in 7 + 8 as (7 + 7) + 1 and the doubles minus one may be used in the same example 7 + 8 as (8 + 8) – 1. This also links to addition of even and odd, odd and odd and even and even numbers. That rule provides a partial check of an answer.

The key number bonds for 10 can be extended to the decimal equivalents starting with the decimal number bonds for 1.0 such as, 0.2 + 0.8. A common error in retrieving these facts may occur when counting in tenths, where the sequence is given as ...0.7, 0.8, 0.9, 0.10 (zero point ten) and, for the previous example, 0.2 + 0.8 = 0.10. This extension of known facts may avoid that error. More place value examples.

Extension to examples such as 6.2 + 3.8 (decimal number bonds for 10) follow the same pattern as for 100 and 1000.

The visual image for these decimal skills could be money. Pound coins are used to represent whole numbers. So, for example, tenths can be represented by trading a £1 coin for ten 10p coins. These can be divided into two lots, demonstrating the number bonds for ten extended to number bonds for 1 with the pattern displayed on the board or screen, maybe using 0.50 + 0.50 as a key reference fact



A second level of splitting up the £1 is then shown by taking one 10p coin and trading it for ten 1p coins. These 'tradings', or 'renamings', are reinforcing the concept of breaking down ones into tenths and hundredths, so that there are now nine 10p coins and ten 1p coins, making £1. The two lots of coins can now be split, first the nine 10s then the ten 1s and the process used to demonstrate examples such as
$0.62 + 0.38 = 1.00 = 1.0 = 1$
Finally, the ten 1p coins can be used to demonstrate examples such as
$0.07 + 0.03 = 0.10 = 0.1$
(7 hundredths plus 3 hundredths equals ten hundredths, and ten hundredths are one tenth).

There needs to be a note of caution here. Earlier we used a 10p coin to represent 10 and a 1p coin to represent 1. This change (inconsistency) needs to be explained before embarking on the demonstration. The same caution applies if base ten blocks are used to represent decimals, where, for example the 100-square now represents one tenth and the 1000 cube represents 1.

The approach in this section provides good opportunities to build on and extend from key basic facts and thus help pupils understand the inter-relationships of numbers and place value, which is an essential skill for mental arithmetic. It should also improve number sense by linking and relating numbers and number facts instead of seeing them in isolation. The visual images help.

5. The addition square

The same demonstration can be used for both the addition and the multiplication facts in order to illustrate progress through the task and the effectiveness of learning key facts and inter-relating them to new facts. The starting challenge is 121 facts on the square, which reduces as the easy facts and links are mastered. A square can be shaded in to show progress:

Facts learnt	Number of facts left to learn
The whole square. Starting off.	121
Adding on zero 0	100
Adding on 1 and 2 (finger counting)	64
Adding on 10 (place value pattern)	49
Adding on 9 (add on 10, subtract 1)	36
Number bonds for 10	31
Number bonds for 10 +/- 1	21
Doubles	16
Doubles +/- 1	10

What facts are left?

5 + 3 and 3 + 5 (which relate to 4 + 4)

7 + 5 and 5 + 7 (which relate to 6 + 6)

8 + 6 and 6 + 8 (which relate to 7 + 7)

8 + 4 and 4 + 8 (8 + 2 + 2)

8 + 5 and 5 + 8 (8 + 2 + 3)

6. Understanding addition and subtraction

It seems that for many learners that addition is the default operation. If all else fails, they add. Subtraction is often avoided, or the procedures/ algorithms are only partially remembered and/ or inaccurately carried out. Counting on is not often used for subtraction. Way back in time (my childhood) the counting on strategy was how shopkeepers made change from purchases.

Data for correct answers from the standardised test in 'More Trouble with Maths':

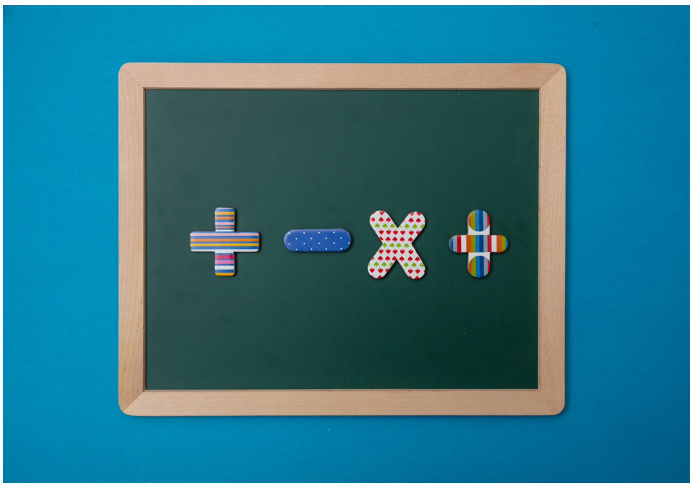
56 +54	10y	92.7%	13y	92.7%
33 - 17	10y	54.5%	13y	72.8%

Vocabulary and language are often a good place to start, since these are the obvious essentials for communicating with learners. The vocabulary of addition and subtraction is quite varied with, for example, 'more', 'plus', 'and', 'in all', 'altogether', and 'increase' are all used to infer +. Later on in the curriculum, in word problems, devious examiners and worksheet writers may well devise word problems where, for example, 'more' may mean subtract in the context of the question. There is a further comprehension factor in that some phrases have other meanings outside numeracy, for example 'take away'. Thus, the vocabulary is both varied and unreliable (see also Chapter 6) and often not used consistently within a school, where, for example, one teacher may use 'renaming' and another 'trading' for the same procedure.

Addition is the next step from counting on in ones. Subtraction is the next step from counting back in ones. If these counting skills are not secure, and that will include crossing the tens, then addition and subtraction may not be understood.

Many children, and adults, consider subtraction to be 'harder' than addition. The roots of this opinion probably lie in the emphasis given to the procedures/algorithms used when subtraction was taught at school (in my day it was 'borrow and payback', later it was 're-grouping' and 'renaming') when what was needed was to understand the procedure and its relationship to addition. Somehow the early experiences some children have of 'taking away' never result in understanding, or generalisations, or the link to addition.

These topics can be built on an understanding of the two operations and their inter-relationship. The links between the two should be mutually supportive. If addition is easier than subtraction that may be down to the fact that we practice counting-up far more than we practice counting-down. And reversing procedures can be a problem. We could address this initially by working from the strength and teach how to subtract by counting on.



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Materials, such as base ten blocks could be used to demonstrate counting back and crossing the tens.

Pupils can be shown how to be flexible with numbers, for example by using the commutative law (7 + 93 = 93 + 7) and the associative law (17 + 8 = 17 + 3 + 5 or 15 + 2 + 8). They can learn the link between addition and subtraction (8 + 7 = 15 and 15 - 8 = 7) so that these facts are connected in their learning experiences. Many supportive strategies depend on such links. The principle is to use the relationships between facts (and operations) to aid learning, for example, 2 + 6 is more efficiently counted as 6 + 2 and that 'bridging' through 10 helps as knowing that 8 + 7 is also 8 + 2 + 5 or 7 + 3 + 5. As above, there is the further link with 8 + 7, and the doubles, 8 + 8 and 7 + 7.

In some curricula the structure is to master addition first then tackle subtraction, but this may not link these two operations in the mind of the learner. Addition and subtraction need to be perceived of as mutually inverse procedures if flexible approaches to mental arithmetic and to checking answers to written problems are to be developed later. For example, as shown above, subtraction can be done and/or checked by adding.

7. Mental calculation strategies for addition and subtraction

These skills build on the previous section and rely very much on those pre-requisite skills and facts being securely fixed in the pupils' memory, and retrievable from memory or by strategy, quickly. This uses up less working memory. Once again, the expectation of speed of response may create additional anxieties in some learners and this in turn may have a negative impact on working memory. On the positive side, automaticity in accessing these facts will leave more working memory to deal with the problem. There is no doubt that a weak working memory will have a disastrous effect on the capacity to perform mental arithmetic calculations. It could be claimed that mental arithmetic discriminates against children with learning disabilities since working memory is frequently a problem in this population. Teachers need to know which of their students have poor working memories.

As well as being able to access basic facts, pupils should be aware of and use the relationship between addition and subtraction. Some simple demonstrations with the number line, moving forward for addition and backwards for subtraction and then discussing the significance of the difference between two numbers and alternative subtraction procedures such as counting on, and emphasising the stepwise nature of this through, for example, 10s and 100s (crossing the tens, and so forth) may help.

As an example: 1000 - 648, where 648 is used as the start point:

648+2=650 .... 650+50=700 ...... 700+300=1000 ...... ANSWER 352

Each of those steps uses the number bonds for 10 (as 10, 100 and 1000), but demands on working memory still remain.

Working on numbers near to tens is another useful extension of key basic facts into mental arithmetic, for example, the use of 10 for 9, 20 for 19, 100 for 98 and other similar examples. The procedure of adding 19 by adding 20 and subtracting 1 uses the inverse link of addition and subtraction as does the subtraction of 19 by using subtract 20 add 1. Base ten blocks and/or money could be used to show the equivalence of adding, for example, 9 one pence coins one at a time against adding a ten pence coin then taking away 1 one pence coin.

Although this mental arithmetic skill is relatively easy to learn, the most likely confusion is that the pupil will do the wrong adjustment, so in 19 + 78, after adding 20 they might add 1 instead of subtracting 1. This is an indication that number values and relationships are not well understood. The question at the intermediate step, 'Is the answer bigger, smaller or the same (than adding 20)?' is useful yet again as a first estimate and check. A second check is from knowing that 9 + 8 gives a ones digit of 7. Learning such checking strategies is essential for accurate mental arithmetic, though time allocations do not always encourage these strategies.

Much can be done by using the inter-relationships between numbers to make mental arithmetic more accessible to more pupils. Pupils will benefit from looking at the numbers before selecting a method rather than reacting solely to the operation symbol. This is another situation where over-viewing is a good technique and another example of the negative effect of demanding speedy reactions. However, some pupils will only be secure if they have one universally applicable method, so adopting this preliminary overview will not be a natural behavior.

Mental arithmetic sessions need to be handled empathetically if some students are not to continually fail and then develop a sense of learned (taught)helplessness and withdrawal from any future involvement in these questions.

Written addition (and subtraction) is traditionally done from right to left, from ones digits to tens digits, hundreds digits and so on. If this method is transferred to mental addition, then the digits in the answer are generated in reverse order, that is the ones digit is calculated first, but stated last in the digits that make up the answer. If pupils add from right to left, starting with the highest place values first, then they can repeat the digits of the answer as they construct the answer, in the correct order. For example, adding 57 and 67 could start at 50 + 60 = 110, then 7 + 7 = 14, taking the answer to 110 + 14 = 124. The strategy of performing mental arithmetic by using the written algorithm /procedure 'in your head' demands a strong working memory.

So, it is likely that there will be a need to encourage and develop appropriate flexibility in approaches for addition and subtraction, not least to recognise that this will happen with some pupils due to different cognitive styles within the pupils that make up any class (see Chapter 4).

8. Paper and pencil procedures for addition and subtraction

These topics could be used to reinforce a range of mental arithmetic methods by encouraging pupils to document procedures other than just the traditional algorithm. There will be some pupils who compute so rapidly and intuitively that documentation will not be easy for them. A classroom ethos which encourages discussing different methods will help. On the other hand, there will be pupils who will be confused by exposure to too many alternatives. The teacher's knowledge of an individual pupil's cognitive style will enable him or her to balance and manage this. The question 'How did you do that?' usually diagnoses the method used. Another positive contributor to this aspect of classroom ethos is that pupils feel confident that their contributions will be valued.

There is significant evidence, from at least two major studies, one in New Zealand (Hattie, 2009)3 and one in the USA (the National Research Council)1 which states that metacognition, that is understanding how you think, has a significant, positive impact on learning.

'Carrying, decomposition, re-naming, regrouping, trading' all refer to an essential procedure. These processes which involve crossing the tens, hundreds, thousands, and so forth boundaries are a development on from counting in ones and require an understanding of place value.

Crossing up, say from ones to tens and crossing back, from tens to ones are complementary and should be demonstrated together in order to reinforce the understanding of these related processes. If subtraction and addition are perceived of as being opposite versions of the same procedure, then these two contributors to the operations of addition and subtraction are also equal opposites.

Of course, crossing the boundaries happens when counting in ones. As addition and subtraction develop to be in numbers that are greater than ten the principle is the same and if the learner is not secure in their understanding of this principle when counting forwards and backwards, they do not have the necessary pre-requisite skill for the next stage of arithmetic.

The relationship between addition and subtraction can be demonstrated with coins or base ten blocks. I prefer coins for older age groups as they have some reality and the trading that happens when the place value boundaries are crossed makes everyday sense. An ever-growing factor that might change my opinion is the use of Touch Cards. So, for now at least, set up an addition with coins alongside the written numbers, say 57 + 78. Add the 1p coins to obtain 15p. Trade ten 1p coins for one 10p coin and carry it across to the tens column. Now add five 10p coins to seven 10p coins and add in the carried 10p coin to make thirteen 10p coins. Trade ten 10p coins for one one-pound coin (100p) and 'carry' it to the hundreds column ... answer 135. Make sure that each step with the coins is recorded as numbers written on paper. The same process can be carried out with base-ten blocks.

Now reverse the procedure and subtract 78 from 135, which will require trading in the £1 coin for ten 10p coins and trading a 10p coin for ten 1p coins. This renames the 135 from 100 + 30 + 5, to 120 + 15. Do some other examples to support understanding and generalizing of this procedure. It is worth demonstrating and discussing each step first rather than the whole procedure in one go. This should also show the relationship between crossing the tens boundaries up and down.

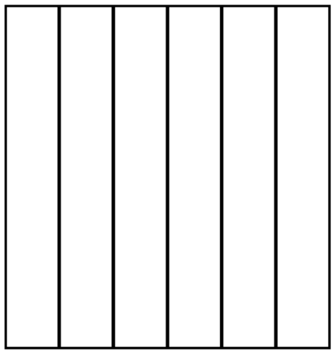
9. Understanding multiplication and division

The four operations, + - x and ÷ are closely inter-related. A clear understanding of each operation and how it relates to the others will strengthen the understanding of all four operations. Multiplication is often described as 'repeated addition' but the understanding of this phrase may not be clear. It should be 'repeated addition of the same number' to be clearer. Similarly, division can be described as 'repeated subtraction' (of the same number).

More data for correct answers from the 'More Trouble with Maths' 15-minute test:

$2\overline{)38}$	10y 59.1%	13y 60.7%	15y 75%
$\begin{array}{r}541\\ \times\underline{203}\end{array}$		13y 15.2%	15y 38.2%

Students will need some experiences with concrete materials to start to develop an understanding of these concepts. As ever, the symbols should be used alongside the manipulatives and visuals used for demonstrations and discussions (see also Chapter 9). As an example, to illustrate 6 x 7, six Cuisenaire rods, value 7, could be introduced, one at a time as 7 + 7 + 7 + 7 + 7 + 7 is written. The six rods could then be placed together to make a rectangle, thus relating to an area model for multiplication. There are, as ever, language considerations as well. The array is 'six lots of seven' and 'six times seven'. The rods can be moved to chow that 6 x 7 can be perceived of as 5 x 7 + 1 x 7. The addition demonstration can then be reversed to demonstrate division as repeated subtraction showing that 42 ÷ 7 gives as an answer of 6, the dimensions of the other side of the rectangle.



The basic ideas in this section of work are the commutative property of multiplication (8 x 7 = 7 x 8), the distributive law {23 x 45 = (20 + 3) x 45 = (20 x 45) + (3 x 45)}, the non-commutative nature of division and that division is the inverse of multiplication. So often a good lesson is about, 'What else are you teaching?'

The connection between addition and multiplication can be explained by starting with times table facts (see Chapter 5) and then linking them to division facts. Whenever possible, a new concept or principle should be introduced and demonstrated with number facts that are automatic for the students.

Area is a good visual aid which can also help in developing the skill of estimating answers. This can be presented with rectangles drawn on paper, squared or plain, and base ten blocks. (See also Chapter 5). Area comparisons can be used to show, whether or not, an estimate is close to the actual answer.

9. Multiplying by zero and by 1

Two simple concepts are introduced here. They may not have been taught to a child in enough detail because they appear deceptively simple.

Multiplying by 1 does not change the value of a number. It is also useful to remember this when renaming fractions. Language could be a factor here. 'One lot of seven' is understandable whereas the more abstract, 'One times seven' is less so, unless you know the maths vocabulary meaning for 'times'. This particular operation was a significant stumbling block for many of the participants who took part in the standardisation of my basic fact tests. The confusion may be in part due to the fact that we rarely multiply by one in everyday maths.

Multiplying by zero results in zero. Many children believe there is a number somewhere, so big that it can over-rule this law. Zero causes many errors! It's an important part of the concept of place value, so it's back to basics again to address this problem.

'Remainders'

The concept of a remainder is useful. There is a possibility that children may otherwise think that division always results in a whole number answer. It also raises a difference between multiplication facts and division facts. Learners may be asked, 'How many fives in fifteen?' which is a division fact directly related to a basic multiplication fact. The question, 'How many fives in sixteen?' is not a division fact that directly relates to a multiplication fact. It is close to a division fact, but the answer now is, 'Three, remainder one.' I am less keen on the phrase, 'Three and one left over' since some children may be confused by the meaning of the 'three' (fives) and the 'one' which is 'left over'. Is that 'one' a 'one five'? This is another example where appropriate materials and visual images would help to clarify the meaning.

10. Rapid recall of multiplication and division facts (see also Chapter 5)

I have, for many years, been asking teachers at my lectures and seminars, 'What percentage of 10-year old pupils do not know all the times table facts?' The answer is becomingly increasingly large, with some teachers saying 80%.

Then there is the word 'rapid'. I can understand that 'rapid recall' may be of benefit when doing mental arithmetic. If these facts can be retrieved quickly with automaticity, then there will be more working memory capacity left to do any calculation. However, the expectation of rapid recall will create anxiety in many students. In turn that anxiety may depress working memory capacity. In my 2009 survey of maths anxiety in secondary school children, 'Having to do maths quickly' was one of the top ranked items for creating anxiety. This situation will probably generate reduced motivation and put a significant number of older pupils into the 20% who fail to reach the required standards in maths (a percentage that did not change for decades and a percentage that I suspect was an underestimate). It seems unproductive to let this happen, since an inability to learn these facts does not preclude success as a mathematician. However, this might have an effect on a pupil's success in school, especially if maths is taught with this as a dominant belief and culture. Your classroom management of this objective will have a profound effect on your examination statistics. Alternative strategies are discussed in this book.

An over-emphasis on rote learning of the times tables, even if addressed with the powerful 'self-voice echo' technique that I researched with Dr Colin Lane (www.self-voice.com) in the 1980s and an under-emphasis on the complementary division facts should be avoided.

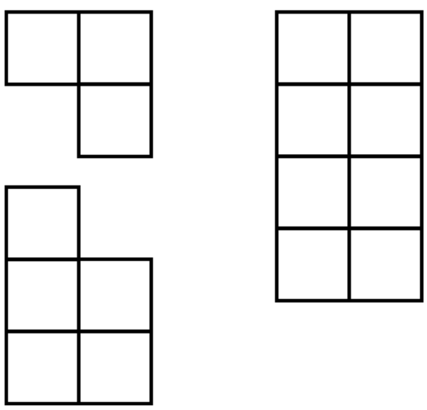
11. Even and odd numbers

This may seem to be a low level of challenge topic, but there are other lessons to learn, as ever. Even numbers end, that is, their ones digit is, 0, 2, 4, 6 or 8. Every other digit in a number can be odd, it's that last one that matters, for example, 975 312 is an even number. 246 827 is an odd number. Any even number is exactly divisible by 2.

Odd numbers end (have a ones digit) in 1,3,5,7 or 9. Every other digit in the number can be even, it's the last one that matters, for example, 864 287 is an odd number. When odd numbers are divided by 2 there is a remainder of 1.

The rules for adding odd and even numbers can act as a low stress (partial) check on the answers to basic addition facts, for example, 7 (odd) plus 8 (even) add to give an odd number (7). And 9 (odd) plus 5 (odd) add to make an even number (14).

Establish (or re-establish) the main idea that even numbers are about 2, about being exactly divisible by 2, using this as an early exposure to the concept of division, sharing into equal parts, halves. And establish that an even number plus 1 makes an odd number. This can be demonstrated with Stern or Numicon materials and visual images and thus lead into discussions of the concept.



One of the useful strategies that pupils may adopt is the breaking down of numbers into easier chunks. For example, doubling 72 might be difficult in one 'bite', but twice 60 plus twice 12 may be easier for some learners. 60 + 12 is a more creative split than the place value split of 70 + 2. The same is true when halving. Half of 60 plus half of 12 as 30 + 6 may be easier than half of 70 plus half of 2. Chunks can encourage flexibility, instead of always splitting according to place value and can help to check answers by using another 'split'.

Again, for example, half of 920 might look daunting, but half of 800 plus half of 120 may be easier for some. 800 + 120 is a more creative split for computing than the place value split of 900 + 20. These decisions require a good sense of number.

Many pupils read numbers literally and in an upwards sequence. For example, 'nine is nine', the number after eight rather than one less than ten. 25 is only seen as a number in the twenties, not as 1/4 of 100 or 1/2 of 50 or even as 20 + 5. Ninety-eight, 98 is seen as nine tens and eight ones and not as 2 less than 100. Can your pupils be taught to find the easy number breakdown? This is a very useful mental arithmetic skill, especially in life where so many prices use a nine as the final digit in a number (139.9p per litre for petrol, £12.95 for a meal in a restaurant, £399 for a laptop). As ever, visual images will help many pupils. For example, a 100 number line may help pupils see the closeness of numbers in the nineties to 100. Coins could also be used. One hundred 1p coins, organized into a pattern of rows of ten, show the closeness of, say 98 to 100.

12. Fractions, decimals and percentages (see also Chapter 10)

Inter-relating these three ways of representing numbers less than one (and bigger of course) will reinforce the understanding of each format. Keep referring to the common equivalents using them to provide 'markers' and to illustrate other examples:

1/2 = 0.5 = 50%, 1/4 = 0.25 = 25%, 1/10 = 0.1 = 10%,

3/4 = 0.75 = 75% (= ½ + ¼ = 0.5 + 0.25 = 50% + 25%)

Data for correct answers from the 'More Trouble with Maths' 15-minute test:

$\frac{4}{7} + \frac{2}{7}$	13Y	63.4%	15Y	73.3%
$\frac{2}{5} + \frac{3}{8}$	13Y	24.6%	15Y	27.3%

13. Using decimal notation for tenths and hundredths

Pupils will be familiar with money written as £3.49. This can be used to give an image of 1/10 as 0.1 and 1/100 as 0.01. Base ten blocks can be used to provide a proportional model. If pupils are having difficulty, show each decimal with money and base ten blocks. Add on coins or blocks, 0.1 and 0.01 to make new numbers. Discuss the place value, as base ten, coins and symbols (digits).

There can be a language and order/direction confusion here for some pupils. Whole number digits progress from the decimal point, right to left, as ones, tens, hundreds, getting bigger, whilst decimal digits go from left to right from the decimal point as tenths and hundredths, getting smaller and with only a slight change in the sound of the words.

Note that the symmetry in decimal numbers is around the ones digit and not about the decimal point.

15. Relating fractions to their decimal representation

Start by focusing on the key related values of 1/2 and 0.5 (It may help some pupils to discuss 0.5 and 0.50). There is another directional difference with whole numbers, 0.5 and 0.50 are the same value whereas 5 and 50 are not. the whole number, 05 is rarely used though it is sometimes used on forms for months, 05 is May or in 24 hour timetables for example, 07.30 (and that decimal point is not really a decimal point!) Other key values are 1/4 and 0.25, 1/10 and 0.1, and 1/00 and 0.01. Set up a table and start to fill in some gaps, such as 0.2 which is 1/5 (and is not 1/20). Show how decimals can be combined, such as 0.3 and 0.5 to make 0.8 and compare the straightforward nature of this, without calculations, to combining fractions such as 3/10 and 1/2. Calculations for fractions to decimals can be shown by divisions with a calculator, especially if patterns can be demonstrated (100/2 = 50, 10/2 = 5, 1/2 =0.5 and 100/10 =10, 10/10 = 1 and 1/10 = 0.1)

16. Understanding percentages

Data for correct answers from the norm-referenced test in 'More Trouble with Maths':

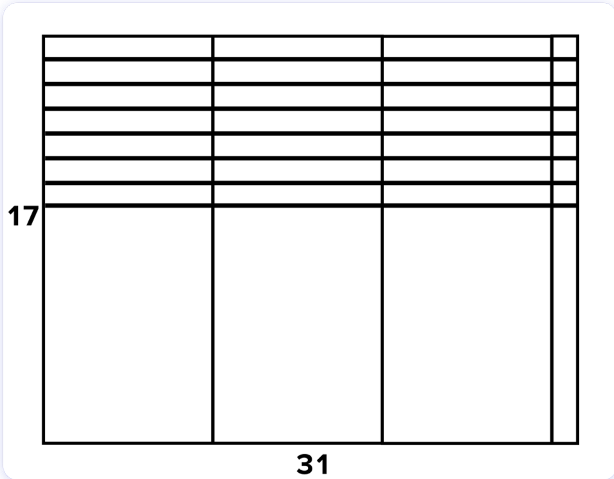
20% OF 140	10Y	32.3%	13Y	53.9%	15Y	66.8%
150% OF 64	10Y	16.4%	13Y	46.1%	15Y	60.0%

Percentages are the third way of representing numbers between 0 and 1 and bigger than 1. Some pupils do not realise, for example, that 200% is 2x. Work from 100% as 1, through 50% as 1/2 and 0.50, 25% as 1/4 and 25%, 10% as 1/10 and 0.01 to 1% and being 1/100 of something. Some pupils may need a brief revision of dividing by 10 and 100. If students can understand that 1% is 1/100 and that it is obtained by dividing by 100 and that 2% is obtained by multiplying the 1% value by 2, that 3% is by multiplying by 3, that 4% is.... and so forth, then they have the foundation for calculating percentages by formula. Calculating core value percentages is a further use for the skill of inter-relating numbers (and estimating). Fill in the gaps on a number line for percentages using core values by discussion such as, where will 50% and 20% go? What fraction and decimal is this value equivalent to? Is 20% twice 10%? Is 5% half of 10%? Do some simple calculations on '25% of' by halving 50% and do 5% calculations by halving 10% values. Combine 25% and 50% to make 75% and 5% and 10% to make 15%. A 100 square is good for visualising and inter-relating percentage values.

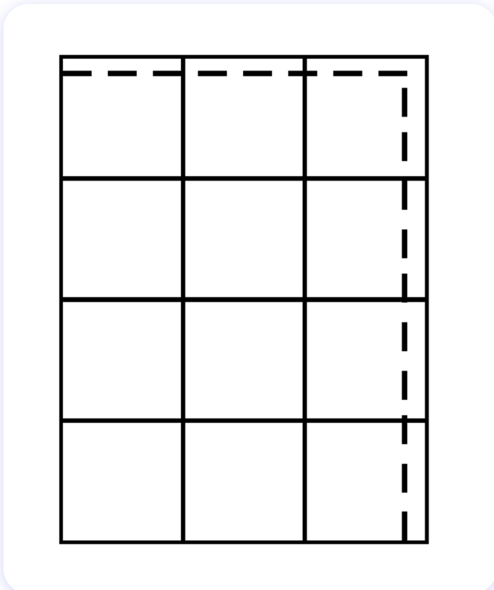
17. Pencil and paper procedures (x and ÷)

Approximations for tens-ones x tens-ones computations are helped by reviewing the area model for multiplication. Some examples are given below

31 x 17 can be seen, as a first estimation, to be over 300, just by counting the hundred squares. A second look would suggest an answer closer to, but less than 600



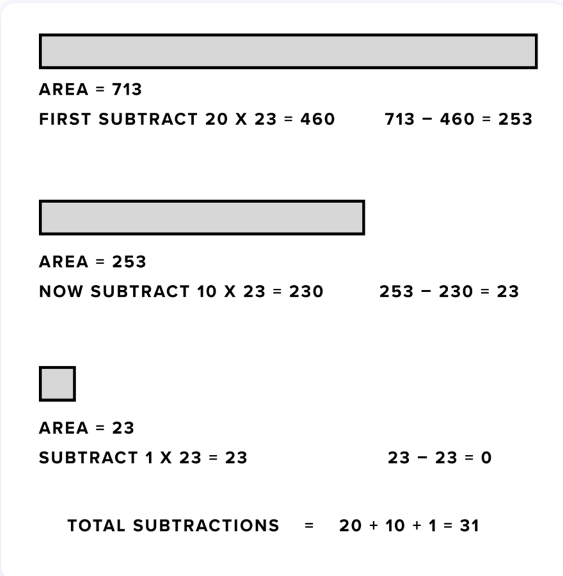
28 x 39 can be seen in comparison to 30 x 40, giving an estimate of less than 1200.



The area model also acts as a good illustration of partition methods. It also emphasizes, place values, the ones, tens and hundreds.

The long multiplication procedure is likely to be one of the first procedural barriers for pupils. It can appear to be abstract, so memory has less chance of a 'realistic hook' to hang from and the organisation of the various written stages, including the 'carried' numbers is challenging. The area model, shown in base ten blocks, or on squared paper or just as sketched rectangles shows where each part of the calculation originates. The area model is also valuable in fractions and algebra. This means the pupils are getting a consistent and developmental image. If you start with the area and one side, then division leads to finding the value of the other side. So, the same image works for multiplication and division and reinforces awareness of the inverse relationship between these two operations.

Division by repeated (chunked) subtraction is the inverse of multiplication by adding chunks (partial products). It can also be shown on the area model, as with Fig. 3.7 for 713 ÷ 23



I like to use the partial products created when setting up key multiplication values, that is 1x, 2x, 5x, 10x, 20x, 50, 100x and so on, using a pattern. So, for example, to divide 1537 by 18, 1537 ÷ 18, set up the table using, and relating, the place value patterns, for example, 5 x 18 = 90 and 50 x 18 = 900

1 X 18 = 18
2 X 18 = 36
5 X 18 = 90
10 X 18 = 180
20 X 18 = 360
50 X 18 = 900
100 X 18 = 1800

The answer to 1537 ÷ 18 can be seen to lie between 50 and 100, closer to 100. This is an initial estimate and is revealed as an integral part of this method. this method provides estimates. Subtraction of multiples will take the pupil to an answer.

1537	
-900	50 x 18
637	
-360	20 x 18
277	
- 180	10 x 18
97	
- 90	5 x 18
7	**Answer : 85 remainder 7**

This does relate to the standard written method. Both are by step by step subtractions of multiples or partial products of (in this case) 18. Obviously, subtraction skills are a pre-requisite.

Checking calculations is usually most effective when a different method is used for checking. Pupils who have flexible approaches to procedures are likely to be much better at checking and evaluating their answers. Flexible styles are explained in Chapter 4.

17. Solving problems

Some problems could be presented where the pupils are not actually required to work out the accurate answer. For example, they could be asked to estimate an answer. This could be as basic as, 'Is it bigger?' or 'Is it smaller?' Pupils could be asked which operation they would use, +, -, x or ÷ and to explain how and why. This could also lead to useful discussions and comparisons of methods and thus to metacognition. The vocabulary around the four operations is varied in content and interpretation. The English language provides several ways of inferring add, subtract, multiply and divide (see Chapter 6). This can also be acknowledged and practised.

Making up 'number stories' can an important activity. Too often teachers expect pupils to 'translate' word problems into mathematical equations/statements whilst forgetting the reverse translation. By doing this, pupils can learn how word problems are constructed (usually resulting in too many totally boring and unrealistic questions in text books) and how misleading features can be introduced, such as extraneous data. It can also create fun and creative discussions!

Making up number stories can help pupils understand how key words can be used to mean different operations and move them away from an overly literal interpretation of vocabulary. (See Chapter 6)

18. Reasoning and generalising about numbers or shapes

It is not always easy for pupils to explain their reasoning for a mental calculation. It will help this objective if the classroom ethos is open and flexible. Even then some pupils may find that their method is so intuitive (and quick) that they cannot really explain all that happened in the brain. This may improve as pupils become accustomed to the idea of analysing their thinking.

Of course, once a teacher knows the procedure favoured by a pupil, she or he might be tempted to suggest changes or alternatives. This may not always be the best move and instant change may well not be possible for the pupil. This whole area of metacognition is fascinating and important if incorporated empathetically. Ideally pupils should learn to be flexible in their choice of methods, be able to successfully use a range of procedures and discuss what they are doing and why. For most pupils this will take a period of time of exposure to the idea and from encouragement to work in this more open manner. It must not be assumed that pupils can adjust their cognitive style overnight.

(For more details on cognitive style see Chapter 4).

This topic area can be used to develop further flexibility in using numbers and operations and to show the inter-relationships, especially those which make the manipulation of numbers easier, for example 49 x 30 calculated via 50 x 30. Again, the language and vocabulary is pertinent, in this case it could be '49 times 30' or '49 multiplied by 30' or '49 lots of 30'.

Spatial examples can be a break from number crunching activities and may well allow some pupils who have strong spatial skills to succeed.

Angle work enters a new world where a key value is 90, not 100, and where the length of the two lines which meet to make an angle do not affect the size/value of the angle.

There are ample examples of angles around us in everyday life which can be used to set the scene for this section. Again, it is possible to build on what the pupil knows, but may not yet have internalised or related. Right angles abound and it is easy to show aspects of two, three and four right angles. An analogue clock face is a good source for 360o, 30o, 90o, 180o, 270o and so on. Diagonals across a square show 45o. There are core values for angles, too.

19. Problems involving 'real life', money and measures

A maths programme should provide ample opportunity for reviews, revisits and revision. Over-learning is a strong, constructive factor in the acquisition of numeracy skills. Additionally, the interlinking of different sections can be used to help develop and consolidate concepts.

Presenting concepts and facts in a 'problem' format, that is, as word problems, possibly with real life content, should be an exercise in developing an understanding of concepts and developing problem solving skills.

Teachers can use these topics to introduce some truly 'real-life' work, such as money, exchange rates and measures. It would seem an ideal section in which to use manipulative materials such as coins, bottles, scales and such. Let pupils experience 1g, 100g, 100ml, 1litre and see some every-day, recognisable examples to give them a basis for judging their answers in this topic. Maybe discuss deceptive packaging used in supermarkets where the height is emphasised at the expense of cross sections.

For foreign exchange it would be good to have some foreign currency, and possibly discuss which values of coins and notes are chosen and why. (For example, the UK works on 1, 2, 5, 10, 20, 50, 100 etc. The US has 'quarters'). This can be a good topic for discussing estimations.

When dealing with recipes, talk about the reality of proportions when calculations may lead to 5 ½ eggs. Perhaps do some costing for recipes. With questions on time, remind pupils that 60 and 12 are the key numbers. Practise counting through a minute and an hour …. 58 seconds, 59 seconds, 60 seconds (which is also1 minute). Then ….. 58 minutes, 59 minutes, 60 minutes, which is also 1 hour. It's the time equivalent of renaming.

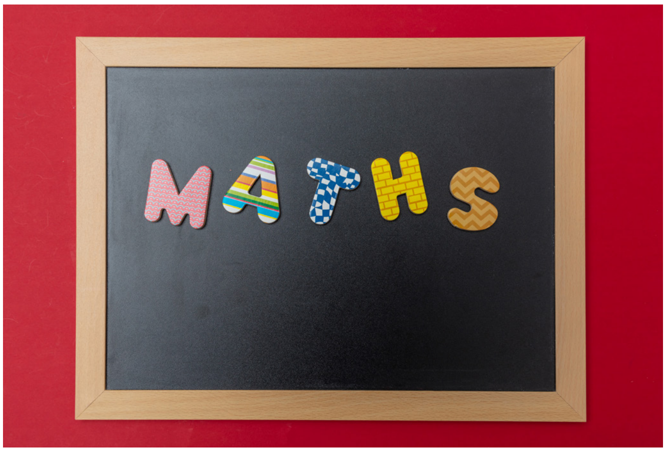
There can be some conceptual problems with the 24-hour clock. It does not use the familiar base ten. The two most likely confusions are with 20:00 hours (8pm) and 22:00 hours (10pm), so try to pre-empt the difficulty. The clock is now the only (moderately) regular base-12 experience we have. Show and explain the use of : instead of . as in, for example, 00:05 and 0.05.

20. Handling Data

Some students who struggle with number can experience success in these topics. It is worth including in an intervention programme some of these topics when this is the case. It's not much fun to always face work that you find difficult. Experiences of success are usually motivating.

21. Probability

This is a topic which lends itself to discussion around events which are within a learner's experience and from which the mathematical groundwork can be naturally derived. Probability is 'everyday', covering topics such as the chance that it will rain to the probability of it being an Eastenders' night.



It allows involvement of all pupils and is an area of maths which, in the introductory stage, is not always a matter of producing an exact answer to be correct. In this introductory stage, pupils can get a feel of probability values (and perhaps a more rational understanding of risks that are often overstated). As ever, it allows for some cross linking to other topics.

22. Organising and interpreting data

Collecting and classifying data is usually a less stressful and less judgmental activity. With careful instruction most pupils should produce acceptable work in this area. The word 'frequency' may cause some confusion and needs good and clear definition.

The advances in computer programmes means that the construction of pie charts, bar graphs and so forth can be done for the pupil thus circumventing the drawing problems some pupils may exhibit, for example, pupils with dyspraxia/developmental coordination disorder. Alternatively, pupils could be given support by supplying a partially completed graph, say with the axes already drawn and labelled, or a data sheet with the chart already drawn and ready for the pupil to use for collecting data.

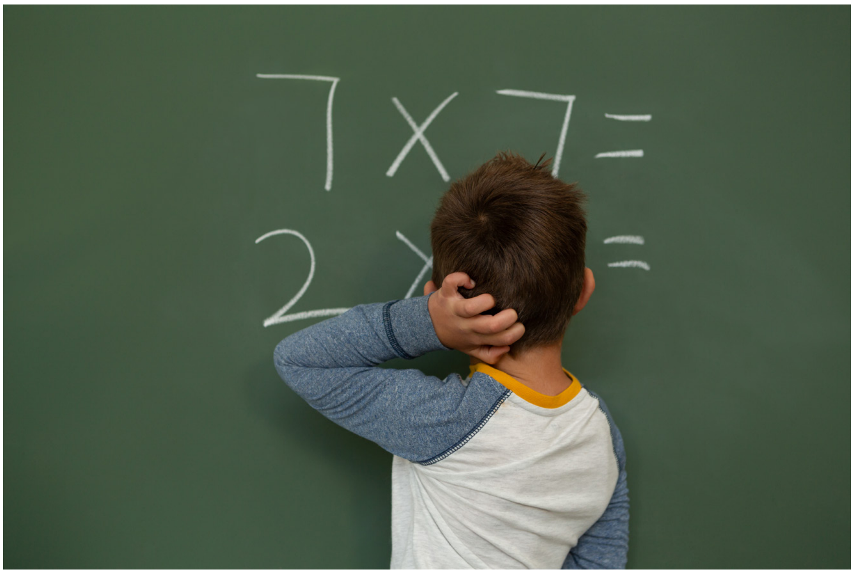
There are sets of data that can be collected which allow the involvement of all pupils, a whole class activity, for example, the colours of cars or vans passing by the school, the heights of pupils, shoe sizes, dates of birth (the day of the month), particular words in a newspaper, comics/magazines, popular sweets and so on.

For line graphs pupils need to know the significance of starting an axis at a value other than zero (and how this can distort the relative values of data ... a qualitative link to proportion). The labelling of axes is another habit that pupils need to acquire.

This section also acts as an early experience of averages as a 'central' measure and an opportunity to evaluate data objectively. This should again give enough opportunities for realistic inclusion.

24. Measures, shape and space

The shape and space section of a curriculum could reveal a new group of pupils who have strengths in these topics in maths and another group which find these topics and concepts more difficult.



25. Measures

This section allows ample scope to work using pupils' existing experiences and to bring together experiences to create understandings and concepts. For example, pupils know the standard soft drink can size and can be shown that this is close to 1/3 of a litre. This fraction can then be shown in terms of cl and ml. The contents of a drinks can can (!) be measured exactly and the result used for discussion on averages and the place of 'precise' and 'approximate' measurements in everyday life. The new work can be built around everyday experience and previous numeracy knowledge, combining revision and awareness in order to develop understanding.

Basic relative values, such as 10mm = 1 cm, 1000mm = 1m and 1000m = 1 km need to be experienced with 'real life' examples as well as just presenting them as symbols to be memorised.

The metric prefixes of m, c, d, and k are not always perceived to be consistent. It can be confusing that m can be used for metre, milli, mile and minute. The kilogram is an anomaly as a standard unit compared to the metre and the litre (in its use of kilo). Also, the kilometre is often viewed as a separate unit to the metre, since the scale and use of this unit is not necessarily connected in everyday perceptions to the relatively small scale, classroom-sized metre. The development of an appreciation of the size of the quantities represented by these units may well need to be addressed explicitly. It is both important and extremely useful to know that the metric prefixes are consistent in that context. (I do feel that examiners and writers of exercise/text-books sometimes inadvertently exacerbate this problem by setting deliberately confusing questions using mixtures of units).

The confusions which might arise in this section can be reduced by using the ample opportunities available to show real items which relate the theory to experience and give learners a baseline, reference image for measures.

Reading from scales is an important, cross-curriculum skill and estimating a reading which falls between divisions is a good estimation skill which relates back to numbers and the number system and the empty number line. Using a large scale (that is with big distances between divisions) could help. Circular and curved scales should also be demonstrated. This also offers some revision of proportion.

Areas may need a fundamental revision before moving to the new work. Perceptions need to be adaptable, for example, it always impresses me that 7 x 7 is an area very close to half of 10 x 10. Area requires new estimation skills. This shift in comparison parameters is demanding and may well be helped by building up some 'easy' areas with base ten 'flats' and cubes. It can also be used to discuss square numbers. Areas can also be used to show the link between multiplication and division. The general formula y = ab is prevalent across maths.

More trouble with maths

Perimeter and area can be confused during calculations, so it is helpful to establish a clear picture in students' minds for each word. A simple verbal image link such as perimeter fence (fence suggests a line) may suffice. It is also useful to emphasise the units used and to create a visual image of a square centimetre and a square metre, and inter-relate them. The use of π and the understanding that it is a constant, not a dimension can be challenging when working on circles.

Reading from timetables requires tracking skills. Pupils who do not have this skill will need structured, small step instruction to learn the skill. A rule or an L shaped piece of card may help with the actual process of tracking. Most rail companies have summary timetables giving only two or three destinations. These offer easier tracking tasks.
A number square or times table square could be used to practise tracking.

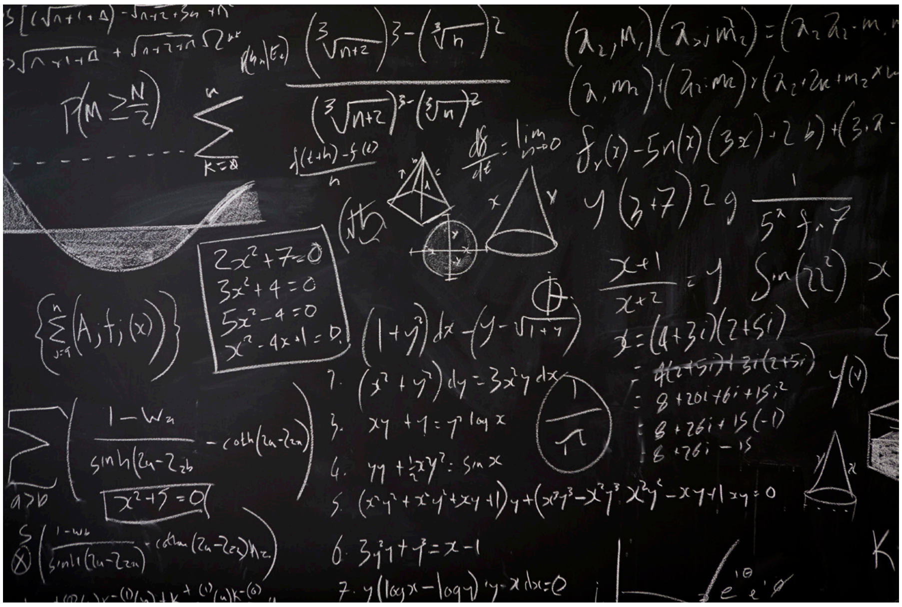
26. Shape and space

The language content around shapes is quite complex. There are some benefits in words like scalene and isosceles in the sense that they are not used in other contexts and with other meanings. There is a considerable new vocabulary to learn and some help may be needed, for example explaining that ‘iso’ means ‘same’ and that ‘octo’ refers to ‘eight’ as in, for example, octopus. There is consistency in the use of these pre-fixes, for example, hex and 6.

Some children will find two-dimensional representations of three-dimensional shapes difficult. Explicit instruction, based on real shapes can help as can isometric paper.

For coordinates, the most likely error will be in mixing up x and y coordinates. The simple mnemonic ‘along the corridor (x) and up the stairs (y)’ may help give the correct order.

Shape and space is a visual topic and must be accompanied by visual teaching materials. Development of this skill should be via hands-on materials, for example, pupils should have nets which they can handle and shape before they advance to doing this purely by using visualisation.



More trouble with maths

About the presenter and author

Steve Chinn

Steve's experience spans over forty years of work and research in special mainstream education. In 1986, he founded, developed and built a specialist secondary school for dyslexic boys, a school which won major national awards. He is now an independent consultant, researcher and writer and has presented papers, contributed to conferences worldwide (over 30 countries) and delivers training courses for psychologists, teachers, parents and learning support teachers.

Maths Explained is a series of instructional online, on-demand video tutorials created by Steve Chinn to help students with dyscalculia and mathematical learning difficulties. It covers

- Understanding numbers
- Place value
- Integers
- Basic facts Addition and Subtraction
- Multiplication
- Division
- Decimals
- Fractions
- Algebra
- Area and Perimeter
- Ratio

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Creates building blocks

The videos explain maths from the beginning, developing an understanding of and confidence in using numbers and maths.

Supports memory

The videos have a developmental structure and inter-linked so that understanding supports memory. This is not about a dependence on rote learning.

Research based

They are underpinned by Steve's research and experience and on research from leading experts from around the world.

Learn at your own pace

The videos, whole or part (you may not need to watch all of a particular video at a stage in your learning, as some parts will be looking ahead to more advanced work), can be watched repeatedly until understanding is secure. No one will say, 'I've already explained that to you too many times'.

Illustrates concepts simply

The videos use simple and clear visual images, often animated, to illustrate the concepts.

Foundation to build on

The videos are not a quick-fix compendium of 'tricks' and mnemonics. They build and link knowledge and constantly reflect back on the roots of maths.

Unlocks learning barriers

They are designed to address and circumvent the learning barriers that prevent so many people from succeeding in maths. There is attention to the details that assist learning.

No age range

The videos are not age-specific (for example, there are no dancing clowns or elephants). Any age of learner can benefit from these videos.